ON FUZZY NEURON MODELS

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Abstract

In recent years, an increasing number of researchers have become involved in the subject of fuzzy neural networks in the hope of combining the strengths of fuzzy logic and neural networks and achieving a more powerful tool for fuzzy information processing and for exploring the functioning of human brains. In this paper, an attempt has been made to establish some basic models for fuzzy neurons. First, several possible fuzzy neuron models are proposed. Second, some learning (training) and adaptation mechanisms for the proposed neurons are given. Finally, the possibility of applying non-fuzzy neural networks approaches to fuzzy systems is also described.

Keywords: Neural networks, fuzzy logic, fuzzy neural networks, learning and adaptation, fuzzy systems.

1. Introduction

A typical neural network has multiple inputs and outputs which are connected by many neurons via weights to form a parallel structure for information processing. The potential benefits of such a structure are as follows: First, the neural network models have many neurons or computational units linked via the adaptive weights arranged in a massively parallel structure. This structure is believed to be essential for building systems with a faster response and a higher performance than the modern sequentially arranged digital computers. In fact, the structure is built after biological neural systems in the hope of emulating and taking advantage of the capabilities of human brains. Second, because of its high parallelism, problems with a few neurons do not cause significant effects on the overall system performance. This characteristic is also called fault-tolerance. Third, probably, the biggest attraction of the neural network models is their adaptive and learning ability. Adaptation and learning are achieved by constantly modifying the weights and other elements. The ability to adapt to and to learn from the environment means that only a minimum amount of information about the environment is needed and that the changes in system characteristics can be adapted.

Neural network models mainly deal with imprecise data and ill-defined activities. The subjective phenomena such as memories, perceptions and images are often regarded as the targets of neural network modeling. It is interesting to note that fuzzy logic is another powerful tool to model phenomena associated with human thinking and perception. In fact, the neural network approach merges well with fuzzy logic [1] and some research endeavors have given birth to the so-called 'fuzzy neural networks' which are believed to have considerable potential in the areas of expert systems, medical diagnosis, control systems, pattern recognition and system modeling.

The term 'fuzzy neural networks' (FNN) has existed for more than a decade. However, the recent resurgence of interest in this area is motivated by the increasing recognition of the potential of fuzzy logic, some successful examples, and the belief that both fuzzy logic and neural networks are two of the most promising approaches for exploring the functioning of human brains. Recently, an increasing number of researchers have become involved in the area of fuzzy neural networks. Yamakawa and Tomoda [2] described a FNN model and applied it successfully to a pattern recognition problem. Kuncicky and Kandel [3] proposed a fuzzy neuron model in which the output of one neuron is represented by a fuzzy level of confidence and the firing process is regarded as an attempt to find a typical value among the inputs. Kiszka and Gupta [4] studied a FNN described by the logic equations. However, no specific learning algorithms are given in these three cases. Gupta and Knopf [5] proposed a fuzzy neuron model which is similar to the first two cases except that a specific modification scheme was proposed for weights adaptation during learning. Nakanishi et al [6] and Hayashi et al [7] used the non-fuzzy neural networks approach for the design of fuzzy logic controllers with adaptive and learning ability. Similarly, Cohen et al [1] used non-fuzzy neural network learning techniques to determine the weights of antecedents for use in fuzzy expert systems. However, no fuzzy neuron models were used.

Some new fuzzy neuron models are proposed in this paper which would overcome some of the weakness of the models mentioned above. Improvements have been made by further and more reasonable modifications of non-fuzzy neuron models into fuzzy ones and, more importantly, by adding learning algorithms to fuzzy neuron models. A class of weighting operators and aggregation operators are proposed which are also called 'synaptic' operators and 'somatic' operators, respectively. Basically, two kinds of fuzzy neuron models are discussed. One is the

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'fuzzification' of non-fuzzy neuron models. The other is where the input-output relations are described by 'If-then' rules. Learning algorithms are proposed for both types of fuzzy neurons.

2. Basic Models of A Fuzzy Neuron

Figure 1 shows the most popular non-fuzzy neuron model proposed by McCulloch and Pitts [8] more than 40 years ago. The neuron has N inputs which are weighted and then passed on to the node. The node sums the weighted inputs and then transfers the results to one of the three nonlinearities. A neuron has an internal threshold level, a predetermined value θ and the neurons fires when the sum of the weighted inputs exceeds the value θ. A mathematical representation of such a neuron is given by:

$$y = f\left(\sum_{i=0}^{N} x_i w_i - \theta \right)$$

where y is the output of the neuron, f represents one of the three nonlinearities, x_i and w_i are the i-th input and its corresponding weighting factor, respectively.

The fuzzy neuron is designed to function in much the same way as the non-fuzzy neuron does, except that it reflects the fuzzy nature of a neuron and has the ability to cope with fuzzy information. The basic structure of a fuzzy neuron is described in Figure 2. The inputs to the fuzzy neuron are fuzzy sets $X_1, X_2, ..., X_n$ in the universes of discourse $U_1, U_2, ..., U_n$, respectively. These fuzzy sets may be labelled by such linguistic terms as high, large, warm, etc. The inputs are then 'weighted' in ways much different from those used in the non-fuzzy case. The 'weighted' inputs are then aggregated not by the summation but by the fuzzy aggregation operations. The fuzzy output Y may stay with or without further operations depending upon specific circumstances. It is also noted that the procedure from the input to the output may not necessarily always be the same.

In the following, detailed discussions of three types of fuzzy neural network models are given. Some possible learning (training) schemes are also proposed.

2.1. A Fuzzy Neuron Described by Logical Equations

In knowledge based systems, one often uses a set of conditional statements, 'If-then' rules to represent human knowledge extracted from human experts. Very often this knowledge is associated with uncertain and fuzzy terms. Therefore, antecedents and consequents in the 'If-then' rules are treated as fuzzy sets. The first fuzzy neuron model we discuss here is described by such rules . In Fig.3, a fuzzy neuron with N inputs and one output is shown and the input-output relations are represented by one 'If-then' rule:

If X_{1i} *and* X_{2i} *and* ... X_{ni} *then* Y_i

here $X_1, X_2, ..., X_n$ are the current inputs and Y_i the current output of the i-th neuron which is described by the i-th rule of the overall M rules shown in Fig. 4. This means that each neuron represents one of the M 'If-then' rules.

According to the fuzzy logic theory, the i-th fuzzy neuron can be described by a fuzzy relation R_i, for example

$$R_i = X_{1i} \times X_{2i} \times ... \times X_{ni} \times Y_i$$

or in the general case:

$$R_i = F(X_{1i}, X_{2i}, ..., X_{ni}, Y_i)$$

where F represents an implication function.

Given the current inputs (fuzzy or non-fuzzy) $X_1, X_2, ... X_n$, according to the compositional rule of inference, the i-th rule gives an output as

$$Y_i' = X_1 \circ (X_2 \circ (... \circ (X_n \circ R_i)...)$$

where o represents any composition operation, such as sup-T-norm.

Here, the proposed fuzzy neuron is the one whose inputs are related to its outputs by a fuzzy conditional statement or a 'If-then' rule. The experience of the neuron is stored in a fuzzy relation R_i, and its output is composed from the current inputs and the past experiences R_i. Therefore, it seems that this artificial fuzzy neuron behaves in much the same way as a biological neuron does.

It should also be noted that inputs to the neuron can be either fuzzy or non-fuzzy, crisp values are the special cases of the fuzzy ones.

The learning algorithms for this fuzzy neuron may vary depending on the real-world problems. Here some basic considerations are given about how the fuzzy neuron changes itself during learning and adaptation. This goal may be achieved by '*synaptic* 'or '*somatic* ' adaptation. '*Synaptic* ' adaptation means all the inputs are constantly modified and then forwarded to the neuron's body, and the '*somatic* ' adaptation implies modifying the past experience. More detailed discussions are given in the next section.

2.2 A Fuzzy Neuron Given by Direct 'Fuzzification' of its Non-Fuzzy Counterpart.

Unlike the above model, the fuzzy neuron proposed in this section is not described by a 'If-then' rule, rather it is obtained by a direct 'fuzzification' or extension of a non-fuzzy neuron model. Similar to a non-fuzzy neuron, all the inputs (fuzzy or crisp) of a fuzzy neuron are modified by weighting or 'synaptic' operations and then go through an aggregation or 'somatic' operations before giving the final results (either a fuzzy set or a membership value). In the following, two fuzzy neuron models as well as their learning schemes are discussed.

2.2.1. A Fuzzy Neuron with Crisp Inputs

As shown in Fig.5 , this fuzzy neuron has N non-fuzzy inputs, and the weighting operations are replaced by membership functions. The result of each weighting operation is the membership value of the corresponding input in a fuzzy set as shown in Fig.6. All these membership values are aggregated together to give a single output in the interval of [0, 1], which may be considered as the 'level of confidence'. The aggregation process represented by ⊗ may use any aggregation or 'somatic' operator, such as, MIN, MAX and any other T-norm and T-conorm [9]. A mathematical representation of such a fuzzy neuron is described by

$$\mu(x_1, x_2, ..., x_n) = \mu_1(x_1) \otimes \mu_2(x_2) \otimes ... \otimes \mu_i(x_i) \otimes ... \otimes \mu_n(x_n)$$

where x_i is the i-th input to the neuron, $\mu_i(.)$ the membership function of the i-th weight, μ the output of the neuron, and ⊗ an aggregation operator.

2.2.2. A Fuzzy Neuron with Fuzzy Inputs

Figure.7 shows another fuzzy neuron which seems to be very much similar to a non-fuzzy neuron except that all the inputs and the output are fuzzy sets rather than crisp values. Each fuzzy input undergoes a 'synaptic' operation which results in another fuzzy set. All the modified inputs are aggregated to produce a N-dimensional fuzzy set. Because the output is rather complicated, it may go through further operations. It must be noted that, unlike the one in the above , the weighting operation here is not a membership function, instead, it is a modifier to each fuzzy input. As shown in Fig.8 , the fuzzy set X_i is modified into another fuzzy set $X_1^{'}$. The aggression operator ⊗ may be the same as the one mentioned above. This fuzzy neuron is mathematically described as

$$Y = X_1 \otimes X_2 \otimes ... \otimes X_i \otimes ... \otimes X_n$$

$$X_i^{'} = G_i(X_i) \qquad i = 1, 2, ..., n$$

where Y is the fuzzy set representing the output of the fuzzy neuron, X_i and X_i the i-th inputs before and after the weighting operation, respectively, G_i the weighting operation on the i-th synaptic connection.

2.3. Learning and Adaptation Mechanisms

An adaptive neuron usually goes through the learning and adaptation processes in order to improve its performance. This goal is often achieved by weights modification or 'synaptic' modification. In the fuzzy neuron model, in addition to 'synaptic' modification, one may also utilize the 'sometic' modification, which means making modifications to the structure of a neuron's body. The learning and adaptive mechanisms discussed here are applicable to all the neuron models discussed above.

2.3.1. 'Synaptic' Modification

During a learning or 'training' process, a neuron constantly changes itself to adapt and improve its performance. 'Synaptic' modification is one of the scenarios to realize this purpose. In a neuron (fuzzy or not), all inputs are modified by weighting or 'synaptic' operations. In the fuzzy case, however, the weighting or 'synaptic' operations are rather complex.

In the case of the fuzzy neuron I, one may also introduce the weighting operations. If so, the fuzzy neurons I and III are considered as the same as far as the weights modifications are concerned. In both cases, all the weights simply serve as mapping functions which transform or modify each fuzzy input into another fuzzy set, this modification process continues until the training results are satisfactory. The modifications may vary depending on the practical problems. However, in the case of triangular fuzzy numbers, the following cases shown in Fig. 9 may occur. Here, the dotted lines represent the fuzzy inputs before modification and the solid ones after modification. In (a), the 'synaptic' operation is a shifting process. In (b), the width of the triangular fuzzy number changes, and in (c), the shape of the triangular fuzzy number changes. In the case of fuzzy neuron II, the weights are membership functions which transforms numerical inputs into their corresponding membership values as shown in Fig. 6. However, the above three modification schemes can also be used in the modifications of the membership functions of the weights in this case.

2.3.2. 'Somatic' Modification

During a learning or 'training' process, a fuzzy neuron may also change its body's structure rather than modifying its inputs. In the case of fuzzy neuron I, this means changing or updating the past experience, which includes

(i) changing the rules,

(ii) changing the membership functions assigned to the fuzzy terms in the rules, or

(iii) changing the way of representing the rules, for example, various implication functions and aggregation operations may be considered [9].

In the cases of fuzzy neurons II and III, many options are available for the aggregation operator $\otimes$, such as T-norms, T-conorms, etc.

2.4. Non-Fuzzy Neural Network Approaches to Fuzzy Systems: Fuzzy Logic Controllers and Fuzzy Expert Systems

In the above sections, several models of a fuzzy neuron are proposed in the hope of combining the strengths of fuzzy logic and neural networks and achieving a more powerful fuzzy information processing tool. However, it is also possible to use the well-established non-fuzzy neural networks approach to fuzzy system modeling and construction where the neural networks' ability to learn and to be trained is particularly attractive. For example, in fuzzy logic controllers and expert systems, one often represents expert knowledge by the 'If-then' rules. A membership function is assigned to each fuzzy term in the rules. By using the backpropagation learning algorithm of conventional neural networks, the membership functions for fuzzy terms are learned based on the training data. More recently, there is a subject starting to attract researchers, it is fuzzy systems modeling using non-fuzzy neural networks approach where membership functions involved in input and output models are learned either from the 'If-then' rules extracted from experienced human operators, or from the numerical data obtained by data logging experiments.

3. Conclusions

Compared with the conventional neural networks theory which has been successfully applied to various areas concerning information processing, the fuzzy neural networks approach, which is intended to play the same role in fuzzy information processing, is still in its infancy. This paper is an attempt to contribute to the further theoretical development of fuzzy neural networks theory.

In this paper, three types of fuzzy neuron models are proposed. The neuron I is described by logical equations or the 'If-then' rules, its inputs are either fuzzy sets or crisp values. The neuron II, with numerical inputs, and the neurons III, with fuzzy inputs, are considered to be a simple extension of non-fuzzy neurons. A few methods of how these neurons change themselves during learning to improve their performance are also given. Finally, the application of the non-fuzzy neural networks approach to fuzzy information processing is also briefly discussed.

Further research work is underway towards implementing these fuzzy neural models in practical problems.

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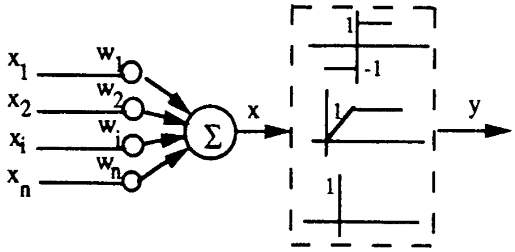

Fig.1 A non-fuzzy neuron model

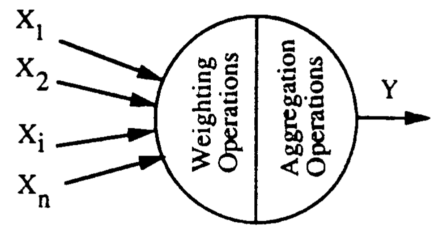

Fig. 2 A fuzzy neuron model

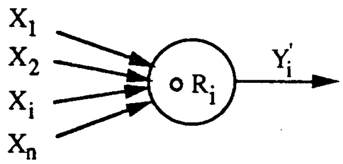

Fig. 3 Fuzzy neuron I

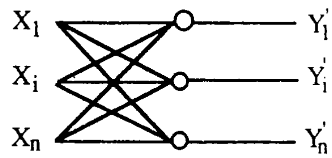

Fig. 4 An example of fuzzy neural networks

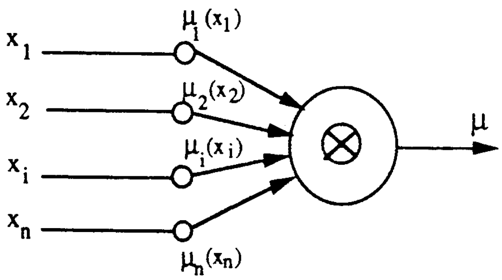

Fig. 5 Fuzzy neuron II

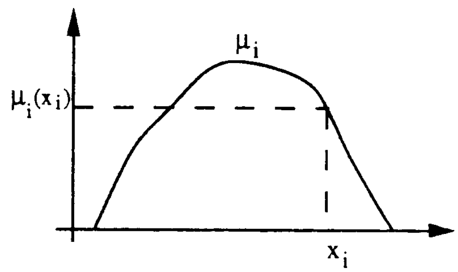

Fig. 6 Weighting of fuzzy neuron II

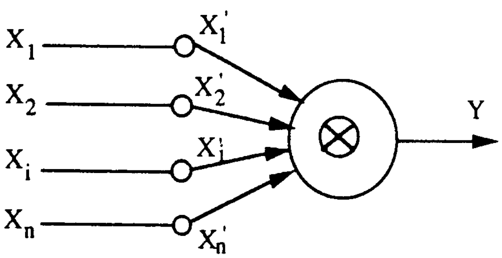

Fig. 7 Fuzzy neuron III

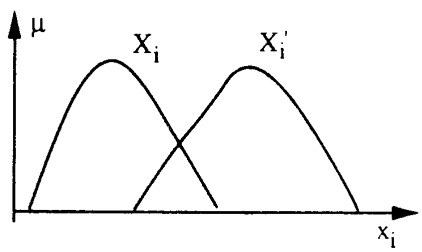

Fig. 8 Weighting of fuzzy neuron III

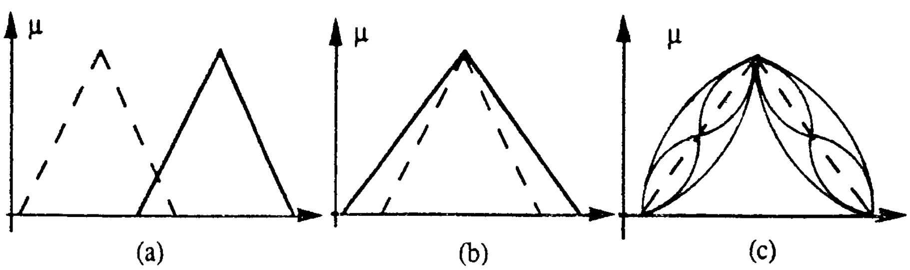

Fig.9 Changes of membership functions during learning